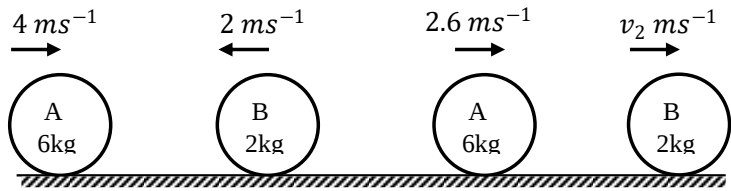
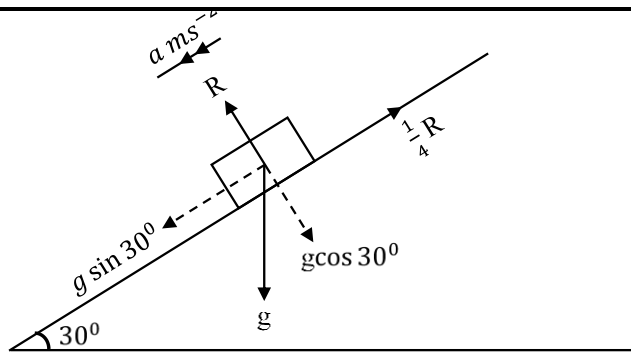


**PROPOSED
MARKING GUIDE
APPLIED MATHEMATICS
P425/2 2023**

NO	SOLUTION	MKS	COMMENT
1	Let X be the number of tails obtained $P(H) = 2P(T)$ $P(H) + P(T) = 1$ $2P(T) + P(T) = 1$ $3P(T) = 1$ $P(T) = \frac{1}{3}, P(H) = \frac{2}{3}$ $\Rightarrow n = 7, p = \frac{1}{3}, q = \frac{2}{3}$ $P(X = 2) = {}^7C_2 \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^5$ $= \frac{224}{729} \text{ or } 0.3073$		
		05	
2	 a) From $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $6 \times 4 - (2 \times 2) = 6 \times 2.6 + 2 \times v_2$ $24 - 4 = 15.6 + 2v_2$ $2v_2 = 4.4$ $v_2 = 2.2 \text{ ms}^{-1}$ b) Loss in kinetic energy = $K.E_{\text{Before}} - K.E_{\text{After}}$ $\text{K.E}_{\text{Before}} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ $= \frac{1}{2} \times 6 \times 4^2 + \frac{1}{2} \times 2 \times 2^2$ $= 52 \text{ J}$ $\text{K.E}_{\text{After}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$		

	$= \frac{1}{2} \times 6 \times 2.6^2 + \frac{1}{2} \times 2 \times 2.2^2$ $= 25.12 \text{ J}$ $\therefore \text{Loss in K. E} = 52 - 25.12 = 26.88 \text{ J}$																													
		05																												
3	$h = \frac{1}{2} - 0 = 0.5$ <table border="1"><thead><tr><th>x</th><th colspan="2">$f(x)$</th></tr></thead><tbody><tr><td>0</td><td>0.1003</td><td></td></tr><tr><td>0.5</td><td></td><td>0.0391</td></tr><tr><td>1</td><td></td><td>0.0801</td></tr><tr><td>1.5</td><td></td><td>0.0602</td></tr><tr><td>2</td><td></td><td>0.0649</td></tr><tr><td>2.5</td><td></td><td>0.0380</td></tr><tr><td>3</td><td>0.0327</td><td></td></tr><tr><td>Total</td><td>0.1330</td><td>0.2823</td></tr></tbody></table> $\int_0^3 f(x) dx \approx \frac{1}{2} \times \frac{1}{2} [0.1330 + 2(0.2823)]$ ≈ 0.1744 $\approx 0.174 \text{ (3dps)}$	x	$f(x)$		0	0.1003		0.5		0.0391	1		0.0801	1.5		0.0602	2		0.0649	2.5		0.0380	3	0.0327		Total	0.1330	0.2823		
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Total	0.1330	0.2823																												
		05																												
4	 Perpendicular to the plane; $R = g \cos 30^\circ \dots\dots\dots(i)$ Parallel to the plane; $g \sin 30^\circ - \frac{1}{4}R = a \dots\dots\dots(ii)$ Putting (i) into (ii); $a = g \sin 30^\circ - \frac{1}{4}(g \cos 30^\circ)$ $a = 9.8 \times \frac{1}{2} - \frac{1}{4} \times 9.8 \times \frac{\sqrt{3}}{2}$																													

$$a = 2.7782 \text{ ms}^{-2}$$

$$\text{From } v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 2.7782 \times 4$$

$$v^2 = 22.2256$$

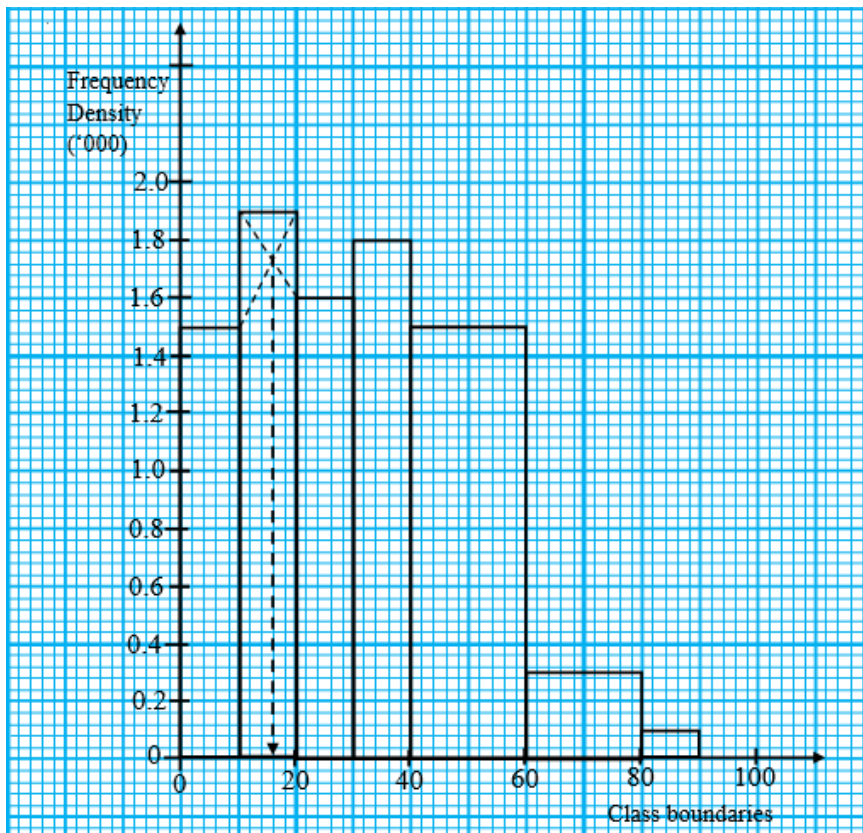
$$\therefore v = 4.7144 \text{ ms}^{-1}$$

05

5

Age (years)	f ('000)	c	$f.d$
0 – 10	15	10	1.5
10 – 20	19	10	1.9
20 – 30	16	10	1.6
30 – 40	18	10	1.8
40 – 60	30	20	1.5
60 – 80	6	20	0.3
80 – 90	1	10	0.1

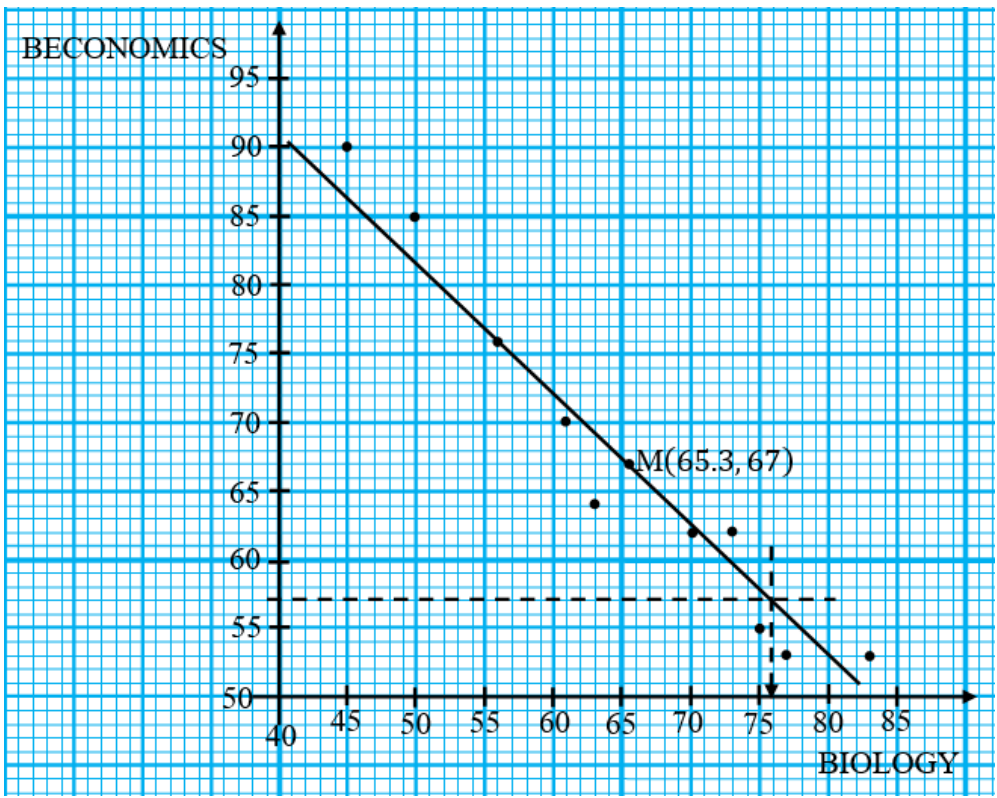
a)



b) Modal age = 16 years

05

6	$x = 6.45, y = 0.00215, z = 2.7$ $e_x = 0.005, e_y = 0.000005, e_z = 0.05$ $w_{min} = \frac{(x+z^3)_{min}}{(\sqrt{y})_{max}}$ $= \frac{6.445+(2.65)^3}{\sqrt{0.002155}}$ $= 539.7147$ $w_{max} = \frac{(x+z^3)_{max}}{(\sqrt{y})_{min}}$ $= \frac{6.455+(2.75)^3}{\sqrt{0.002145}}$ $= 588.4137$ $\therefore \text{Internal} = 539.7147 \leq w \leq 588.4137$																		
		05																	
7	a) From $P(S' \cap R') = P(S') \cdot P(R')$ $P(S) = [1 - P(S)] \cdot P(R')$ $P(S) = \frac{1}{4} (1 - P(S))$ $P(S) = \frac{1}{4} - \frac{1}{4} P(S)$ $\frac{5}{4} P(S) = \frac{1}{4}$ $\therefore P(S) = \frac{1}{5} \text{ or } 0.2$ b) $P(S' \cap R) = P(S') \cdot P(R)$ $= \frac{4}{5} \times \frac{3}{4}$ $= \frac{3}{5} \text{ or } 0.6$																		
		05																	
8	Let K = weight per unit area, $W = KA$ Area of square = $60 \times 60 = 3600 \text{ cm}^2$ Area of circle = $\pi r^2 = \pi \times 20^2 = 400\pi \text{ cm}^2$ Remainder = $3600 - 400\pi$ <table border="1"> <thead> <tr> <th>Figure</th><th>Area</th><th>Weight</th><th>Distance of C.O.G from AD</th></tr> </thead> <tbody> <tr> <td>Square</td><td>3600</td><td>3600K</td><td>30</td></tr> <tr> <td>Circle</td><td>400π</td><td>$400\pi K$</td><td>40</td></tr> <tr> <td>Remainder</td><td>$3600 - 400\pi$</td><td>$(3600 - 400\pi)K$</td><td>$\bar{x}$</td></tr> </tbody> </table> Taking moments about AD	Figure	Area	Weight	Distance of C.O.G from AD	Square	3600	3600K	30	Circle	400π	$400\pi K$	40	Remainder	$3600 - 400\pi$	$(3600 - 400\pi)K$	$\bar{x}$		
Figure	Area	Weight	Distance of C.O.G from AD																
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	$(3600 - 400\pi)K \times \bar{x} = 3600K \times 30 - 400\pi K \times 40$ $(3600 - 400\pi)\bar{x} = 108,000 - 16,000\pi$ $\bar{x} = \frac{108,000 - 16,000\pi}{3600 - 400\pi}$ $\bar{x} = 24.6375 \text{ cm from AD}$		
		05	
9	a) $M(\bar{x}, \bar{y})$ $\bar{x} = \frac{\sum x}{n} = \frac{653}{10} = 65.3$ $\bar{y} = \frac{\sum y}{n} = \frac{670}{10} = 67$ $\therefore M(\bar{x}, \bar{y}) = M(65.3, 67)$  The scatter plot shows the relationship between BIOLOGY (x-axis) and BECONOMICS (y-axis). The x-axis ranges from 40 to 85 with major grid lines every 5 units and minor grid lines every 1 unit. The y-axis ranges from 50 to 95 with major grid lines every 5 units and minor grid lines every 1 unit. There are 10 data points plotted. A solid black line represents the linear regression line. The mean point is labeled M(65.3, 67). Dashed lines extend from the mean point to the axes, and a vertical dashed line with an arrow points down from the mean point to the x-axis at 65.3. b) Let B = BIOLOGY E = ECONOMICS		

B	E	R_B	R_E	d	d^2
45	90	10	1	9	81
63	64	6	5	1	1
56	76	8	3	5	25
61	70	7	4	3	9
75	55	3	8	-5	25
83	53	1	9.5	-8.5	72.225
73	62	4	6.5	-2.5	6.25
50	85	9	2	7	49
77	53	2	9.5	-7.5	56.25
70	62	5	6.5	-1.5	2.225
653	670				$\sum d^2 = 327$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(327)}{10(10^2-1)}$$

$$= -0.9818$$

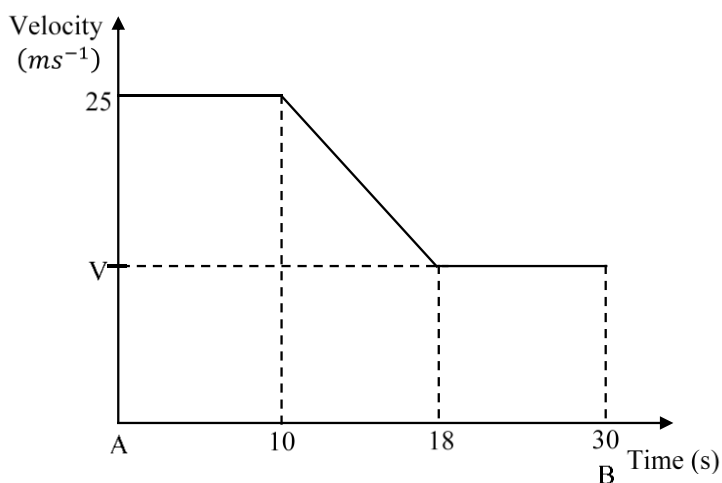
Comment: It is significant at 5%

Or It is significant at 1%

Or It is very high negative correlation

12

10 a)



b)

$$(i) \quad 526 = (25 \times 10) + \frac{1}{2} \times 8(25 + V) + (V \times 12)$$

$$526 = 250 + 100 + 4V + 12V$$

$$526 = 350 + 16V$$

	$16V = 176$ $V = 11 \text{ ms}^{-1}$ (ii) From $v = u + at$ $11 = 25 + 8a$ $8a = -14$ $a = -1.75$ $\therefore$ the deceleration is 1.75 ms^{-2}														
		12													
11	a) (i) $f(1) = 1e^1 + 5(1) - 10 = -2.2817$ $f(2) = 2e^2 - 5(2) - 10 = 14.7781$ (ii) Since $f(1) \cdot f(2) < 0$, thus $1 < \text{root} < 2$ b) <table border="1"> <tr> <td>1</td> <td>x_0</td> <td>2</td> </tr> <tr> <td>-2.2817</td> <td>0</td> <td>14.7781</td> </tr> </table> $\frac{x_0 - 1}{0 + 2.2817} = \frac{2 - 1}{14.7781 + 2.2817}$ $x_0 = 1.1337$ $f(1.1337) = 1.1337e^{1.1337} + 5(1.1337) - 10$ $= -0.8089$ <table border="1"> <tr> <td>1.1337</td> <td>x_1</td> <td>2</td> </tr> <tr> <td>-0.8089</td> <td>0</td> <td>14.7781</td> </tr> </table> $\frac{x_1 - 1.1337}{0 + 0.8089} = \frac{2 - 1.1337}{14.7781 + 0.8089}$ $x_1 = 1.1787$ $\therefore \text{Root} \approx 1.179 \text{ (3dps)}$	1	x_0	2	-2.2817	0	14.7781	1.1337	x_1	2	-0.8089	0	14.7781		
1	x_0	2													
-2.2817	0	14.7781													
1.1337	x_1	2													
-0.8089	0	14.7781													
		12													
12	a) Testing For $-1 \leq x \leq 0$; $= F(0) - F(-1)$ $= \frac{1}{6} - 0$ $= \frac{1}{6} < \frac{1}{2}$ For $0 \leq x \leq 2$; $= F(2) - F(0)$														

$$= \frac{5}{6} - \frac{1}{6}$$

$$= \frac{2}{3} > \frac{1}{2}$$

Let $m = \text{median}$

$$F(m) = \frac{1}{2}$$

$$\frac{1+2m}{6} = \frac{1}{2}$$

$$1 + 2m = 3$$

$$2m = 2$$

$$\therefore m = 1$$

b) For $-1 \leq x \leq 0$; $f(x) = \frac{d}{dx} \left(\frac{1+x}{6} \right) = \frac{1}{6}$

For $0 \leq x \leq 2$; $f(x) = \frac{d}{dx} \left(\frac{1+2x}{6} \right) = \frac{1}{3}$

For $2 \leq x \leq \frac{8}{3}$; $f(x) = \frac{d}{dx} \left(\frac{4+3x}{12} \right) = \frac{1}{4}$

For $x \geq \frac{8}{3}$; $f(x) = \frac{d}{dx} (1) = 0$

$$\therefore f(x) = \begin{cases} \frac{1}{6}, & -1 \leq x \leq 0 \\ \frac{1}{3}, & 0 \leq x \leq 2 \\ \frac{1}{4}, & 2 \leq x \leq \frac{8}{3} \\ 0, & \text{elsewhere} \end{cases}$$

c) $P(1 \leq X \leq 2.5) = \int_1^2 \frac{1}{3} dx + \int_2^{2.5} \frac{1}{4} dx$

$$= \left[\frac{1}{3} x \right]_1^2 + \left[\frac{1}{4} x \right]_2^{2.5}$$

$$= \frac{1}{3} (2 - 1) + \frac{1}{4} (2.5 - 2)$$

$$= \frac{1}{3} + \frac{1}{8}$$

$$= \frac{11}{24} \text{ or } 0.4583$$

d) $E(X) = \int_{-1}^0 \frac{1}{6} x dx + \int_0^2 \frac{1}{3} x dx + \int_2^{\frac{8}{3}} \frac{1}{4} x dx$

$$= \frac{1}{6} \left[\frac{x^2}{2} \right]_{-1}^0 + \frac{1}{3} \left[\frac{x^2}{2} \right]_0^2 + \frac{1}{4} \left[\frac{x^2}{2} \right]_2^{\frac{8}{3}}$$

$$= \frac{1}{12} (0 - 1) + \frac{1}{6} (4 - 0) + \frac{1}{8} \left(\frac{64}{9} - 4 \right)$$

	$= -\frac{1}{12} + \frac{2}{3} + \frac{7}{18}$ $= \frac{35}{36} \text{ or } 0.97222$		
		12	
13	a) Let $\mathbf{F}_R$ be the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ $= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -11 \end{pmatrix}$ $= \begin{pmatrix} 5 \\ -12 \end{pmatrix} \text{ N}$ $\therefore \mathbf{F}_R = \sqrt{5^2 + (-12)^2} = 13 \text{ N}$ b) About the origin; $G = \begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} 3 & -2 \\ -2 & -11 \end{vmatrix}$ $= (-6 - 6) + (-4 - 15) + (-33 - 4)$ $= -12 - 19 - 37$ $= -68 \text{ Nm}$ Using $G - xF_y + yF_x = 0$ $-68 + 12x + 5y = 0$ $\therefore 12x + 5y - 68 = 0$ c) It cuts the x-axis when $y = 0$; $12x - 68 = 0$ $12x = 68$ $x = 5.6667 \text{ m from the origin.}$ d) Let $(a\mathbf{i} + b\mathbf{j})$ be the force to be added to form a couple For a couple the $\mathbf{F}_R = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 5 \\ -12 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $5 + a = 0 \quad \therefore a = -5$ $-12 + b = 0 \quad \therefore b = 12$ $\therefore (-5\mathbf{i} + 12\mathbf{j}) \text{ N}$		
		12	

14

a) Let $x = N^{\frac{1}{k}}$

$$x^k = N$$

$$x^k - N = 0$$

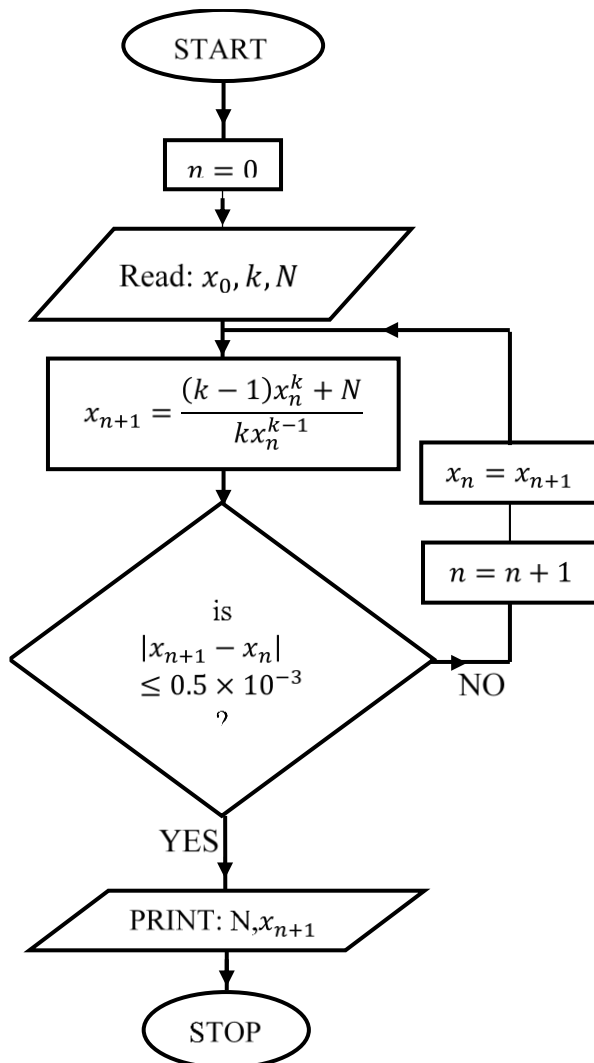
$$\text{Let } f(x) = x^k - N$$

$$f'(x) = kx^{k-1}$$

$$x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}}$$

$$\begin{aligned} x_{n+1} &= \frac{kx_n^k - x_n^k + N}{kx_n^{k-1}} \\ &= \frac{(k-1)x_n^k + N}{kx_n^{k-1}} \end{aligned}$$

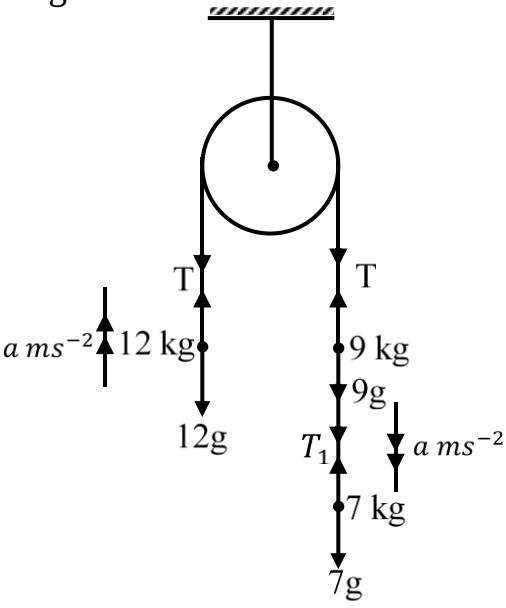
b)



c) Dry run

$$N = 13, k = 4, x_0 = 1.6$$

	<table border="1"> <tr> <th>n</th><th>x_n</th><th>x_{n+1}</th><th>$x_{n+1} - x_n$</th></tr> <tr> <td>0</td><td>1.6</td><td>1.9935</td><td>0.3935</td></tr> <tr> <td>1</td><td>1.9935</td><td>1.9054</td><td>0.0881</td></tr> <tr> <td>2</td><td>1.9054</td><td>1.8989</td><td>0.0065</td></tr> <tr> <td>3</td><td>1.8989</td><td>1.8989</td><td>0.0000</td></tr> </table> $\therefore \text{Root} = 1.899, N = 13$	n	x_n	x_{n+1}	$ x_{n+1} - x_n $	0	1.6	1.9935	0.3935	1	1.9935	1.9054	0.0881	2	1.9054	1.8989	0.0065	3	1.8989	1.8989	0.0000		
n	x_n	x_{n+1}	$ x_{n+1} - x_n $																				
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		12																					
15	a) $P(A) = 0.3, P(A') = 0.7$ $P(B) = 0.5, P(B') = 0.5$ $P(C) = 0.7, P(C') = 0.3$ (i) $P(\text{only one}) = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$ $= (0.3 \times 0.5 \times 0.3) + (0.7 \times 0.5 \times 0.3) + (0.7 \times 0.5 \times 0.7)$ $= 0.045 + 0.105 + 0.245$ $= 0.395$ (ii) $P(\text{two or more}) = P(\text{only two}) + P(\text{all})$ $P(\text{only two}) = P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A \cap B \cap C')$ $= (0.7 \times 0.5 \times 0.7) + (0.3 \times 0.5 \times 0.7) + (0.3 \times 0.5 \times 0.3)$ $= 0.245 + 0.105 + 0.045$ $= 0.395$ $P(\text{All}) = P(A \cap B \cap C)$ $= 0.3 \times 0.5 \times 0.7$ $= 0.105$ $\therefore P(\text{two or more}) = 0.395 + 0.105 = 0.5$ ALT: $P(\text{two or more}) = 1 - [P(\text{none}) + P(\text{only one})]$ $P(\text{none}) = P(A' \cap B' \cap C')$ $= 0.7 \times 0.5 \times 0.3$ $= 0.105$ $\therefore P(\text{two or more}) = 1 - [0.105 + 0.395]$ $= 1 - 0.5$ $= 0.5$																						

	b) $P(P) = 0.6, P(L/P) = \frac{2}{3}$ $P(Q) = 0.4, P(L/Q) = \frac{1}{3}$ (i) $P(L) = \left(0.6 \times \frac{2}{3}\right) + \left(0.4 \times \frac{1}{3}\right)$ $= \frac{8}{15}$ or 0.5333 (ii) $P(P/L') = \frac{P(P \cap L')}{P(L')}$ $= \frac{\left(0.6 \times \frac{1}{3}\right)}{1 - \frac{8}{15}}$ $= \frac{3}{7}$ or 0.4286		
		12	
16	Let a be the acceleration of the system, T and T_1 be the tensions in the strings  a) (i) For 12 kg mass: $T - 12g = 12a$(i) For 9 kg mass: $T_1 + 9g - T = 9a$(ii) For 7 kg mass: $7g - T_1 = 7a$(iii) (i)+(ii)+(iii);		

	$4g = 28a$ $a = \frac{4g}{28} = \frac{g}{7} = \frac{9.8}{7} = 1.4 \text{ ms}^{-2}$ (ii) From (i); $T = 12a + 12g$ $T = 12 \times 1.4 + 12 \times 9.8$ $T = 16.8 + 117.6$ $= 134.4 \text{ N}$ From (iii); $T_1 = 7g - 7a$ $= 7 \times 9.8 - 7 \times 1.4$ $= 68.6 - 9.8$ $= 58.8 \text{ N}$ b) Let the velocity be v Using $v = u + at$ $v = 0 + 1.4 \times 1.5$ $v = 2.1 \text{ ms}^{-1}$		
--	---	--	--